Fluency

Day 3

Math & Science Collaborative
at the Allegheny Intermediate Unit

Understanding Multiplication and Division

- What does it mean to understand multiplication?
- Look at the standards for the operations. Read the standards dealing with multiplication and division.
- Then talk with your group. Develop a chart, graphic, or model showing what students need to know to show an understanding of multiplication.

Building Understanding While Focusing on Fluency

- Use models to represent multiplication and division
  - Arrays, set models, area models, number lines
- Use problem contexts/real-life situations
  - Make sure all four categories of problems are addressed
- Productive talk
- Classroom environment

Understanding Multiplication and Division

- Address the big ideas
  - Guide the types of questions that are posed
- Explore symbolic representations
- Use models to represent addition and subtraction
  - Number lines, manipulatives, ten frames, number charts
- Explore concepts through problems and literature

Components of a Math Task

- Developing the Context
- Supporting the Investigation
- Preparing for the math congress
- Facilitating the math congress
- Integrating mini-lessons, games and routines

Components of a Math Task

- **Developing the Context**
  - Can use stories, situations (realistic or fictional), contexts, models
  - Children work to explore and make sense of the situations
  - They try out strategies to solve and make sense of the use of the strategies
  - They explore and generate patterns
  - They generalize
  - And "mathematize"
- Supporting the Investigation
- Preparing for the math congress
- Facilitating the math congress
- Integrating mini-lessons, games and routines

Muffle's Truffles

- Use blank paper to record **your own** thinking about solving the task. *Actually do the work to solve the problem yourself.* Think as a learner.
- Share your solution strategies with your small group

Anticipating

- Actively envision how students might mathematically approach the instructional task or tasks that they will work on
- Involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies – both correct and incorrect – that they might use…and how those strategies relate to the mathematical concepts, representation, procedures and practices…

- *5 Practices for Orchestrating Productive Mathematics Discussions* by Margaret Smith and Mary Kay Stein

Muffle's Truffles

- Do some Anticipating
- Together, think as teachers. Strategize about other solution strategies that students in your grade 3 or 4 classroom might use.
- Record these strategies for yourself.

- Now, examine the student work samples.
- Were you able to anticipate these strategies? If not, no worries. Just by doing the anticipating, you are more prepared to deal with *un*anticipated strategies.

Components of a Math Task

- Developing the Context
- **Supporting the Investigation – AKA: Monitoring**
  - Facilitator observes strategies
  - Listens to discussions
  - Confers with pairs or small groups
  - Ask questions and make comments (not leading ones)
    - Help me understand your method
    - What made you decide to use that strategy?
- Preparing for the math congress
- Facilitating the math congress
- Integrating mini-lessons, games and routines

A word about Differentiation

- Rich and engaging tasks differentiate themselves

Sample student dialogue

- Examine the snip-it of conversation on the Conferring with Students at Work handout. This is from Toni's ***monitoring*** of the students while they work.

  - What is Toni doing to assist his/her students?
  - What isn't Toni doing to assist his/her students?

What is the math in this task?

- Place Value patterns, especially with groups of ten
- Unitizing
- Quotative division – Finding how many groups

Multiplication and Division Word Problem Types

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems

Components of a Math Task

- Developing the Context
- Supporting the Investigation
- Preparing for the math congress
- Facilitating the math congress
- Integrating mini-lessons, games and routines
  - Can be used at the start of a lesson for 10-15 minutes
  - Designed to highlight a particular computational strategy
  - Designed to help build fluency
  - Might help with mental math
  - Make sure to structure the games so that they actually support strategies and discussions, not just fact practice

Components of a Math Task

- Developing the Context
- Supporting the Investigation
- Preparing for the math congress
  - Have children talk with partners about what they want to share (can use posters or paper with doc camera)
  - Might do a gallery walk
  - Facilitator decide what ideas to have shared and what order to have the ideas shared
  - AKA: Selecting and Sequencing
- Facilitating the math congress
- Integrating mini-lessons, games and routines

Components of a Math Task

- Developing the Context
- Supporting the Investigation
- Preparing for the math congress
- Facilitating the math congress
  - NOT a whole class share
  - Designed to push the math development of students
  - Many possible structures to use
  - Make sure to ask questions
  - Help students make connections among ideas, among strategies, among representations, etc.
- Integrating mini-lessons, games and routines

Math Congress

- Examine the sample excerpt from the Math Congress in Toni's class, entitled A Portion of the Math Congress
  - What is Toni doing to assist his/her students?
  - What isn't Toni doing to assist his/her students?

Math Congress

- What would you focus on in the Math Congress for these tasks?
  - First Task
    - Unitizing
    - Place value
    - Connection in this task is the fact that the number of tens (the unit)is the number of full boxes
  - Variation
    - Initial use of distributive property (don't need to name it yet)
    - Commutativity (as related to congruence of arrays)
  - Might move from least to most efficient strategies

Quick Images

- Note that they are still used in grade 3 or 4 (and beyond)
- In this unit, they are being used to build larger arrays

Some general questions

- Who can put what Sarah just said into your own words?
- Who has a question or comment for Daniel?
- Who agrees with Ashley, but used a different strategy?
- Who still needs convincing that Carmine's strategy will work?
- Will Bailey's strategy **always** work? How do you know for sure?

Strings of Related Problems

- Explicitly designed to guide learners towards computational fluency
- Build automaticity **by focusing on <u>relationships</u>**

Strings of Related Problems

2 x 5

4 x 5

4 x 10

10 x 4

10 x 6

6 x 10

10 x 12

10 x 18

11 x 18

Mathematical Models

- Different mathematical models/representations/tools have been documented to have different effects on mental computation
  - With addition and subtraction, the open number line stimulates a mental representation of numbers and operations on those numbers that is helpful for developing mental strategies
  - With multiplication and division, the open array plays a similar role. It strongly reinforces the relationship between partial products and total area, thus the distributive, commutative and associated properties

Mathematical Models

- The Open Array can support the development of:
  - A wide range of strategies including skip-counting, repeated addition, doubling, doubling and halving, partial products
  - Big ideas like the distributive, commutative and associative properties
  - Visual models for what multiplication actually means
  - An understanding of both area and perimeter

Mathematical Models

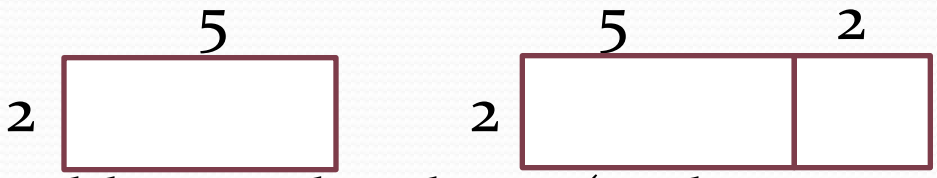
- However, coordinating rows and columns simultaneously is difficult for many.
- Therefore, students need time to understand the array itself as well as its organization
- The time is worthwhile because of the power of the array for thinking about multiplication and division

Mathematical Models

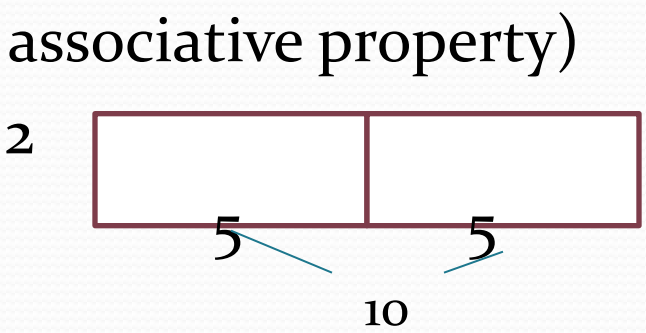
- Model of the Situation
  - Visually representing the situation itself
- Models of Students' Strategies
  - Using the models provides students with a chance to see and discuss each others' strategies
- Model as a Tool for Thinking
  - Use the model to explore and prove their ideas

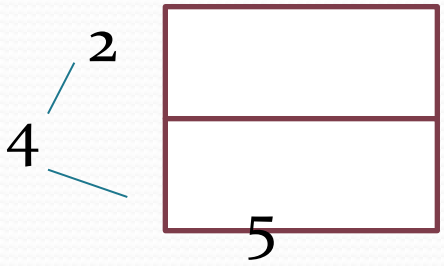
Model as a Tool for Thinking

- Use the model (the array) to explore and prove their ideas
  - Distributive property

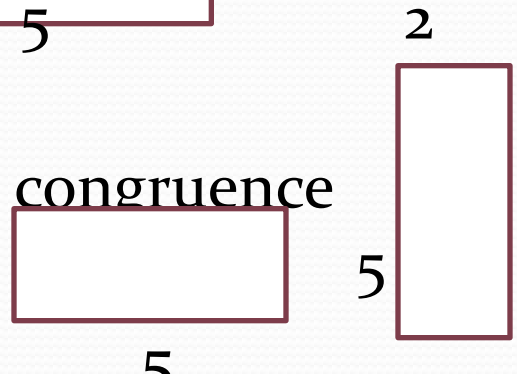


  - Doubling and Halving (and eventually related to the associative property)





  - Commutative Property, equivalence, congruence



You – next steps

- Can we do what we have always done regarding fluency?
- What will you do regarding fluency expectations?
- What might you do differently?
- Beyond the classroom – what about parents – their expectations regarding fluency?